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Ginzburg–Landau equation and vortex liquid phase of Fermi liquid superconductors

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Abstract
In this paper we study the Ginzburg–Landau (GL) equation for Fermi liquid superconductors with strong Landau interactions F_{0s} and F_{1s}. We show that Landau interactions renormalize two parameters entering the GL equation, leading to the renormalization of the compressibility and superfluid density. The renormalization of the superfluid density in turn leads to an unconventional (2D) Berezinskii–Kosterlitz–Thouless (BKT) transition and vortex liquid phase. Application of the GL equation to describe underdoped high-T_c cuprates is discussed.

The understanding of the physics behind high-T_c cuprates remains one of the major challenges to the condensed matter physics community nowadays [1–3]. One big mystery is the pseudo-gap phase in underdoped cuprates, where a d-wave-like gap in the quasi-particle spectrum exists, but the system is already a normal metal [1–3]. It has been suggested that the pseudo-gap phase can be understood as a vortex flow state of a layered superconductor with strong phase fluctuations [4] existing in the temperature range $T_c < T < T^*$ (T_c is the superconductor transition temperature and T^* is the pseudo-gap temperature). To test this idea, the Ginzburg–Landau (GL) action for a strong coupling superconductor corresponding to the physical picture of preformed pairs has been derived and studied [5, 6]. However the GL action is not able to explain the different trends of T_c and T^* as functions of doping x in the pseudo-gap phase, presumably due to the lack of consideration of the effect of proximity to the Mott transition [2–4]. In this paper we shall explore this problem further by including the effect of Landau interactions in the GL action. We shall derive the GL action for a Fermi liquid superconductor with strong density–density and (transverse) current–current Landau interactions F_{0s} and F_{1s}. The GL action we derive is applicable to general Fermi liquid superconductors with strong Landau interactions F_{0s} and F_{1s} and is not restricted to high-T_c cuprates. We shall show how Landau interactions renormalize two parameters entering the GL action, leading to the renormalization of the superfluid density and compressibility. The effects of Landau interactions on superfluid dynamics, including critical magnetic fields and critical current, vortex structures and vortex liquid phase, will be examined. We show that Landau interactions provide a natural mechanism for the separation of the temperature scales T^* and

T_c in the underdoped cuprates. However, many properties of the pseudo-gap phase remain unexplained [2, 3, 7, 8].

In Fermi liquid theory, the long-wavelength and low-frequency electromagnetic response of a Fermi liquid superconductor to external electromagnetic perturbations can be described by an effective Hamiltonian [9]

$$H = H^{\mathrm{BCS}} + \frac{1}{2}\sum_q N_{\mathrm{F}}^{-1}\left(\frac{F_{1\mathrm{s}}}{k_{\mathrm{F}}^2}\vec{j}_t(q)\cdot\vec{j}_t(-q) + F_{0\mathrm{s}}n(q)n(-q)\right) \tag{1}$$

where H^{BCS} is the mean-field Bardeen–Cooper–Schrieffer (BCS) Hamiltonian, and $q = (\vec{q},\omega)$. N_{F} is the density of state on the Fermi surface, k_{F} is the Fermi momentum and $F_{1\mathrm{s}}$ and $F_{0\mathrm{s}}$ are the Landau parameters describing (transverse) current–current and density–density interactions in the system, respectively. We shall not restrict ourselves to translational invariant systems here and therefore there is no particular relation between the Landau parameter $F_{1\mathrm{s}}$ and the effective mass m^*/m. We disregard all other Landau interactions in this paper since their effects on the GL action are much weaker. Integrating out the fermion fields, the (transverse) current and density responses of the system to an electromagnetic field is given by the effective action [9]

$$\begin{aligned} S_{\mathrm{eff}} = \sum_q \Bigg(&\frac{1}{2}\left(\frac{1}{K_0(q;T)} - \frac{F_{1\mathrm{s}}}{(1+F_{1\mathrm{s}})\bar{K}_0(0)}\right)\vec{j}_t^{\,2} \\ &+ \frac{1}{2}\left(\frac{1}{\chi_0(q;T)} - \frac{F_{0\mathrm{s}}}{\bar{\chi}_0(0)}\right)n^2 - \vec{j}_t\cdot\vec{A} - n\phi\Bigg) \\ = \sum_q \Bigg(&\frac{1}{2}\left(\frac{\vec{j}_t^{\,2}}{K_0(q;T)} + \frac{n^2}{\chi_0(q;T)}\right) - \vec{j}_t\cdot(\vec{A}+\vec{a}) \\ &- n(\phi+\varphi) + \frac{1}{2}\left((1+F_{1\mathrm{s}}^{-1})\bar{K}_0(0)\vec{a}^2 + \frac{\bar{\chi}_0(0)}{F_{0\mathrm{s}}}\varphi^2\right)\Bigg) \end{aligned} \tag{2}$$

where T is the temperature and $K_0(q;T)$ and $\chi_0(q;T)$ are the transverse current–current and density–density response functions for the BCS superconductor in the absence of Landau interactions, respectively. $-\bar{K}_0(T) = -K_0(\vec{q}\to 0,\omega=0;T)$ and $-\bar{\chi}_0(T) = -\chi_0(\vec{q}\to 0,\omega=0;T)$ are the corresponding superfluid density and compressibility of the BCS superconductor. $\vec{A}$ and ϕ are the external electromagnetic vector and scalar fields. Fictitious gauge potentials $\vec{a}$ and φ are introduced to decouple the current–current and density–density interactions (Legendre transformation) in the third and fourth line of equation (2). In this representation, the Landau interactions are absorbed by introducing fictitious vector and scalar fields that couple to the BCS superconductor.

The corresponding Ginzburg Landau action is therefore of the form

$$\begin{aligned} S_{\mathrm{GL}} = \int \mathrm{d}^d x \int \mathrm{d}t \Bigg(& -\mathrm{i}\gamma\psi^*(\partial_t + \mathrm{i}e^*(\phi+\varphi))\psi + \frac{1}{2}\frac{\bar{\chi}_0(0)}{F_{0\mathrm{s}}}\varphi^2 \\ &+ \frac{\hbar^2}{2m^*}\left|\left(\nabla - \mathrm{i}\frac{e^*}{\hbar c}(\vec{A}+\vec{a})\psi\right)\right|^2 - \alpha(T)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}\frac{\bar{K}_0(0)}{G}\vec{a}^2\Bigg), \end{aligned} \tag{3}$$

where $G = F_{1\mathrm{s}}/(1+F_{1\mathrm{s}})$, which is the GL action for a BCS superconductor coupling to the effective electromagnetic fields $\phi+\varphi$ and $\vec{A}+\vec{a}$. The dynamics of the fictitious gauge fields are given by the Legendre transformation in equation (2). For weak-coupling BCS superconductors, $\alpha(T) \sim \epsilon(T_{\mathrm{M}}^2/E_{\mathrm{f}})$, where $\epsilon = 1 - T/T_{\mathrm{M}}$, $\beta \sim N_{\mathrm{F}}^{-1}(T_{\mathrm{M}}/E_{\mathrm{f}})^2$, and $\gamma = \gamma' + \mathrm{i}\gamma''$, where $\gamma' \sim (T_{\mathrm{M}}/E_{\mathrm{f}})^2$, and $\gamma'' \sim T_{\mathrm{M}}/E_{\mathrm{f}}$. T_{M} is the mean-field (BCS) transition temperature and E_{f} is the Fermi energy [5]. In the strong coupling limit, the

GL action becomes the Gross–Pitaevski action for a gas of charge $e^* = 2e$ bosons, with $\alpha(T) \to \bar{\mu} \sim -\epsilon E_b/2$ being the chemical potential for the bosons, where $E_b \sim T_M$ is the bound state energy for the electron pair, $m^* \to 2m$, $\gamma \to 1$, and $\beta \to 4\pi a_b/m^*$, where $a_b > 0$ is the (composite) boson scattering length [5]. Consistency between equations (2) and (3) implies that $\bar{K}_0(T) = -(e^{*2}\alpha)/(m^* c^2\beta)$ and $\bar{\chi}_0(T) = -(\gamma_1 e^*)^2/\beta$.

The fictitious gauge fields $\vec{a}$ and φ can be eliminated easily from equation (3). Writing $\psi = \sqrt{\rho}\,e^{i\theta}$, we obtain

$$S_{GL} \to \int d^d x \int dt \Bigg(\rho\gamma(\partial_t\theta + e^*\phi) + \frac{\hbar^2}{2m^*}(\nabla\sqrt{\rho})^2 - \alpha(T)\rho + \frac{\bar{\beta}}{2}\rho^2 + \frac{\hbar^2}{2m^*}\frac{\rho}{(1+G'(\rho))}\left(\nabla\theta - \frac{2\pi}{\Phi_0}\vec{A}\right)^2 \Bigg) \tag{4}$$

where $\Phi_0 = hc/e^*$ is the fluxoid quantum, $\bar{\beta} = \beta(1 + F_{0s})$ and $G'(\rho) = -G\rho(T)/\rho(0)$, where $\rho(0) = \alpha(T = 0)/\beta$.

Equation (4) is the main result in this paper. We observe that the Landau interactions renormalize two parameters in the GL action, with $\beta \to \bar{\beta} = \beta(1 + F_{0s})$ and the superfluid density ρ_s (or London penetration depth λ^{-2}) renormalized to

$$\rho_s(T) = \frac{\rho(T)}{(1+G'(\rho))} = \frac{(1+F_{1s})\rho(T)}{1 + F_{1s}(1-\rho(T)/\rho(0))}, \tag{5}$$

in agreement with result from Fermi liquid theory [9].

We now discuss some general properties of the renormalized GL action. We first consider F_{0s}. F_{0s} simply renormalizes the (inverse) compressibility of the system through $\beta \to \bar{\beta}$ with, correspondingly, $\bar{\chi}_0(T) \to \bar{\chi}_0(T)/(1 + F_{0s})$. Notice that the velocity of the superfluid density fluctuation (Goldstone) mode $c \sim \sqrt{\rho_s/\bar{\chi}_0(T)}$ [9] is renormalized by both F_{0s} and F_{1s} as a result[1]. The transition temperature T_c (governed by $\alpha(T)$) and characteristic (coherence) length $\xi_A^2(T) = \hbar^2/(2m^*\alpha(T))$ governing the gap amplitude fluctuations are not renormalized by Landau interactions. However, the coherence length governing phase fluctuation is renormalized, with $\xi_P(T) \sim \xi_A(T)/\sqrt{(1 + G'(\rho))}$. The length scales governing the amplitude and phase fluctuations separate in the presence of Landau interaction F_{1s}. Notice that the renormalization effects associated with F_{1s} is proportional to $\rho(T)$. Therefore, at $T \sim T_M$ or $H \to H_{c2}$, where $\rho(T) \to 0$, the renormalization effects of F_{1s} become unimportant and the only effect of Landau interaction is renormalization of β.

Next we consider critical magnetic fields and critical currents. The thermal dynamical critical field H_c and the upper critical field $H_{c2} \sim \Phi_0/2\pi\xi^2$ are not renormalized by F_{1s}. The latter is because $\rho \to 0$ at $H \to H_{c2}$, and F_{1s} becomes unimportant. The lower critical field $H_{c1} \sim (\Phi_0/4\pi\lambda^2)\ln(\lambda/\xi_P)$ is renormalized by F_{1s} through both $\lambda = \sqrt{(1 + G'(\rho))}\lambda_0$ and $\xi_P \sim \xi_A/\sqrt{(1 + G'(\rho))}$, where λ_0 is the London penetration depth of the corresponding pure BCS superconductor and ξ_P is the effective vortex core size defined by superfluid density (see below). In particular, for $1 + F_{1s} < 1 \lambda > \lambda_0$, $\xi_P < \xi_A$ and H_{c1} is reduced by F_{1s}.

The critical current passing through a thin wire or film can be obtained by first minimizing the free energy with fixed velocity $\vec{v}_s = \hbar(\nabla\theta - \frac{2\pi}{\Phi_0}\vec{A})$ and then determining the maximum possible supercurrent $\vec{j}_s = e^*\rho_s\vec{v}_s$ [10]. It is easy to see that the critical current is reduced (enhanced) by nonzero $F_{1s} < (>)0$, and that the reduction (enhancement) is stronger at lower temperature. As a result, the rate of reduction of the critical current as the temperature increases is slower (faster) than in the corresponding BCS superconductor when $F_{1s} < (>)0$. The rate of

[1] Notice that with long-ranged Coulomb interaction, the superfluid density fluctuation mode becomes 'gapped' (Anderson–Higgs mechanism) through $F_{0s} \to \frac{4\pi e^{*2}}{q^2} + F_{0s}$.

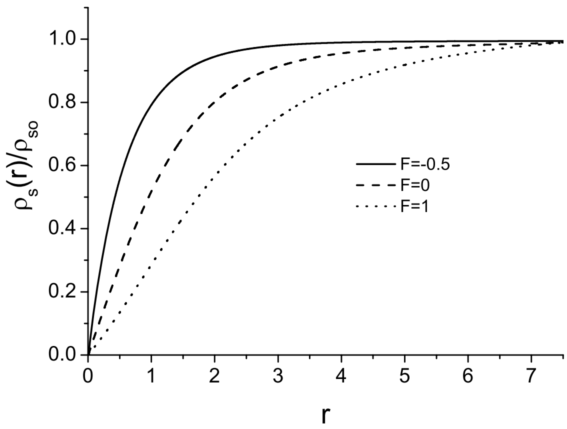

Figure 1. Superfluid density $\rho_s(r)/\rho_{so}(\rho_{so} = \rho_s(r \to \infty))$ as function of r for three values of $F_{1s} = 1.0, 0.0, -0.5$.

decrease can be fitted roughly by the formula $j_c(T) \sim j_c(0)(1 - T/T_M)^\nu$, where $\nu = 3/2$ for BCS superconductors, and changes continuously when F_1 changes. We find numerically that $\nu \sim 1.35$ for $F_{1s} = -0.9$ and that there is an increase to $\nu \sim 1.7$ for $F_{1s} = 0.5$.

Next we discuss vortices. First we consider the single vortex solution of the GL equation, i.e., the solution of the form $\psi(r,\phi) = f(r)\,e^{i\phi}$. We shall consider $T = 0$ for simplicity. The separation of length scale associated with amplitude and phase fluctuations implies that two 'sizes' of the vortex core can be defined. One, $\sim\xi_A$, is the size of the region where the amplitude of the BCS wavefunction $f(r)$ goes to zero. This is not (directly) renormalized by Landau interaction. The other, $\sim\xi_P$, is the size of the region where the superfluid density $\rho_s(r)$ defined by equation (5) goes to zero. The two coherence lengths differ when the Landau interaction F_{1s} is nonzero. To see that ξ_P represents the core size defined by the superfluid density, one may replace the density variable ρ by the superfluid density variable ρ_s in the GL action using equation (5) and derive the corresponding GL equation in terms of ρ_s and θ. Performing a small r expansion for the vortex solution ($\nabla\theta \sim \hat{\phi}/r$), it is straightforward to see that ξ_P represents the coherence length for the superfluid density. In figure 1 we show $\rho_s(r)/\rho_s(r \to \infty)$ as a function of r/ξ_A at zero temperature solved numerically for three different values of $F_{1s} = -0.5, 0, 1$. The dependence of superfluid density vortex core size on F_{1s} is clear.

Vortex viscosity is also affected by Landau interactions. In standard Bardeen–Stephen (BS)-type arguments [10], the vortex flow viscosity η_v is given by

$$\eta_v \sim \left(\frac{a_v}{\xi^2}\right)\sigma_L,$$

where a_v is a constant of order O(1) and ξ, σ_L are the effective vortex core size and normal state conductivity for the Fermi liquid superconductor, respectively. Both ξ and σ_L are renormalized by F_{1s}, with $\sigma_L \sim (1 + F_{1s})^2\sigma_0$, where σ_0 is the conductivity for the normal metal without Landau interactions and $\xi \sim \xi_P$.

The renormalized GL action provides a natural explanation for one problem faced by the ordinary GL action (in both the weak and strong coupling limits) when applied to underdoped high-T_c cuprates—the vanishing of the superfluid density ρ_s when the concentration of holes $x \to 0$ and the huge difference between the temperature T_c and T^* in the pseudo-gap phase. In models of strong correlations, the vanishing of the superfluid density is a direct consequence of the proximity to a Mott insulator and is reflected in the Landau parameter F's, with $F_{1s} \sim x-1$ at $T \to 0$ and F_{0s} remaining regular in gauge theories [2, 3, 11]. The superfluid density ρ_s is renormalized by the same factor $1 + F_{1s} \sim x$ (equation (5)) in the GL action. For weakly coupled layers of two-dimensional superconductors with $x \ll 1$, the renormalization of phase stiffness but not amplitude stiffness in the GL action implies that the temperature T_c, which is determined by the BKT transition [12], and $T^* \sim T_M$, which is determined by the BCS mean-field transition, separates. The BKT transition is determined by ρ_s and happens at a temperature [12]

$$kT_c \sim \frac{\hbar^2}{4m^*}\rho_s \sim \frac{\hbar^2}{4m^*}x\rho(0),$$

which is much lower than the mean-field transition temperature T_M. The amplitude of the gap parameter is only weakly renormalized in this temperature regime because ξ_A is unrenormalized.

We notice, however, that many properties of high-T_c cuprates are not explained by this simple model of a Fermi liquid superconductor, which includes only F_{0s} and F_{1s}. First of all, rotational symmetry is strongly broken in the Fermi surface geometry of cuprates, and the Landau interactions $F_{\vec{k}\sigma\vec{k}'\sigma'}$ depend in general not only on the relative angle between $\vec{k}$ and $\vec{k}'$, but also on the directions of the $\vec{k}$ and $\vec{k}'$ vectors themselves. This is not taken into account in our simple parameterization of the Landau parameters. The strong Fermi surface asymmetry is reflected, for example, in photoemission experiments which probe the Fermi surface structure directly [1, 2]. The model also cannot explain the temperature dependence of the London penetration depth λ in the low-temperature limit [2, 3] $T \to 0$ and the apparent narrowness of the paraconductivity regime [2, 3, 7] above T_c ($\sim$ several T_c, see also [13]) in the pseudo-gap phase. In the vortex liquid phase, the conductivity of the system is given in the two-fluid picture by $\sigma = \sigma_L + \sigma_v$, where σ_L is the quasi-particle (normal) conductivity and σ_v is the vortex liquid conductivity. In the BS picture, $\sigma_v \sim \eta_v/n_v \sim (\frac{a_v}{\xi_P^2})\sigma_L/n_v$, where n_v is the vortex density. Therefore,

$$\sigma \sim \sigma_L\left(1 + \frac{a_v}{\xi_P^2 n_v}\right),$$

and σ_v dominates as long as $\xi_P^2 n_v \ll 1$. At $T \gg T_c$, $\xi_P^2 n_v \sim e^{-\epsilon_c/kT}$, where ϵ_c is the vortex core energy. In GL theory, $\epsilon_c \sim (\hbar^2/2m^*)(\alpha/\beta) \sim E_f \gg T_M$ in the weak coupling limit and there is a crossover to $\epsilon_c \sim T_M$ in the strong coupling limit [3]. Therefore, ϵ_c is as least of the order of $T^* \sim T_M$ and $\xi_P^2 n_v \ll 1$ at temperatures $T \ll T^*$, meaning that paraconductivity should dominate over most of the pseudo-gap regime in this simple GL model. It was proposed that the vortex core energy can be largely reduced if there exists a competing order state close in energy to the superconducting state and the vortex core is in the state of competing order [2, 3]. In this case $\xi_P^2 n_v$ can be of order O(1) at a temperature of order several T_c, resulting in a much narrower paraconductivity regime. We shall not discuss this possibility here since it is outside the scope of the GL action we present in this paper.

Summarizing, in this paper we have studied the effect of the Landau interactions F_{0s} and F_{1s} on the Ginzburg–Landau action for superconductors. We find that F_{0s} renormalizes the parameter β and correspondingly the (inverse) compressibility in the GL action. The effects of F_{1s} on the GL action are much more interesting. It renormalizes the superfluid density and separates the length scale for amplitude and phase fluctuations. For $1 + F_{1s} \ll 1$ it provides a natural mechanism for separation of the mean-field transition temperature ($T_M \sim T^*$) and BKT transition temperature (T_c) and this suggests that the pseudo-gap phenomenon observed in high-T_c cuprates may be a rather general property of Fermi liquid superconductors with strong current renormalization, $F_{1s} < 0$. We point out, however, that our simple model of a Fermi liquid superconductor with Landau interactions F_{0s} and F_{1s} cannot explain many properties of the high-T_c cuprates, and that a much more sophisticated model is needed to describe realistic underdoped cuprates.

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